Destabilizing Divergences in Supergravity Theories at Two Loops

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Abstract

We examine the stability of the mass hierarchy in hidden-sector supergravity theories. We show that a quadratically divergent tadpole can appear at two loops, even in minimal supergravity theories, provided the theory has a gauge- and global-symmetry singlet with renormalizable couplings to the visible fields. This tadpole can destabilize the hierarchy. We also find a quadratically divergent two-loop contribution to the field-dependent vacuum energy. This result casts doubt on the efficacy of the "LHC mechanism" for controlling quadratic divergences. We carry out the two-loop calculation in a manifestly supersymmetric formalism, and explain how to apply the formalism in the presence of supersymmetry breaking to derive radiative corrections to the supersymmetric and soft supersymmetry-breaking operators. Our approach greatly simplifies the calculation and guarantees consistency of our results with the underlying supergravity framework.

1 Introduction

The chief motivation for supersymmetry is that it provides the only field theoretical framework with which to interpret light fundamental scalars. For renormalizable theories, this is due to the fact that supersymmetry prevents quadratically divergent graphs from renormalizing the scalar potential. In this sense, supersymmetry solves the hierarchy problem associated with light scalar fields.

However, it is expected that a realistic theory will be a nonrenormalizable effective field theory, where the nonrenormalizable interactions are suppressed by a scale M. This might be as large as the Planck mass, M_P, as in supergravity theories, or it might be some lower scale where other new physics comes into play. In either case it is important to understand the extent to which nonrenormalizable operators affect the hierarchy problem.

When supersymmetry is not broken, the supersymmetric nonrenormalization theorems ensure that masses are not renormalized in effective theories. The nonrenormalization theorems also protect against the emergence of operators not already present in the superpotential, even those consistent with gauge and global symmetries. This leads to a revised notion of naturalness appropriate for supersymmetric theories.

When supersymmetry is broken, however, the situation is more subtle. One would like to know whether radiative corrections induce divergent operators which destabilize the hierarchy. One would also like to know whether supersymmetric naturalness still holds, and in particular, whether radiative corrections generate all possible soft supersymmetry-breaking operators consistent with the symmetries.

In essence, supersymmetry reduces by two the degree of divergence associated with perturbative diagrams. This is best understood in superspace, where naive power counting automatically incorporates supersymmetric cancellations. The effects of supersymmetry breaking can be summarized in terms of a (chiral) superspace spurion, U, whose F-term contains a supersymmetry-breaking vev. In hidden-sector supergravity theories, U is either the conformal compensator or a hidden-sector chiral field. In either case, $U = M_S^2 \theta\theta$, where M_S is the scale of supersymmetry breaking [1].

In an effective supersymmetric theory, radiative corrections can induce quadratically divergent contributions to the Kähler potential. These include terms of the form

$$\delta K \;=\; \frac{\Lambda^2}{M^2}\, U^+ U \;+\; \frac{\Lambda^2}{M^2}\, U^+ N \;+\; \frac{\Lambda^2}{M^2}\, N^+ U \;+\; \frac{\Lambda^2}{M^2}\, N^+ N \;+\; ... \,, \qquad (1.1)$$

where $\Lambda \simeq M$ is the cutoff and N is an arbitrary chiral superfield. For hidden-sector models, where $M = M_P$ and $U = M_S^2 \theta\theta$, these terms give a contribution of order M_S^4 to the vacuum energy, and

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generate a tadpole of order $M_S^2 N$ in the superpotential. As we will see, such terms are dangerous: they can destabilize the hierarchy and induce masses of order M_S for the scalar fields.

Recently, there has been a controversy about whether radiative corrections generate divergent tadpoles in supergravity theories [2], [3]. In this paper we resolve the question. We show, consistent with the result of Jain [3], that *at one loop*, there are no quadratically divergent contributions to singlet tadpoles in supergravity theories with minimal Kähler potentials. We demonstrate, however, that quadratically divergent tadpoles do appear at two loops, and that these corrections are sufficient to destabilize the hierarchy. Furthermore, we find that tadpoles occur at one loop in models with nonminimal Kähler potentials, where the visible sector is directly coupled to the hidden sector that is responsible for supersymmetry breaking.

The hierarchy can also be destabilized by quadratically divergent contributions to the (field-dependent) vacuum energy. In the so-called no-scale "LHC models" of Ferarra, Kounnas and Zwirner [4], the scale of supersymmetry breaking is undetermined at tree level. Instead, it is fixed by minimizing the one-loop effective potential. This requires a careful cancellation of one-loop quadratic divergences which would drive the scale of supersymmetry breaking to zero or to the cutoff Λ.

It might be argued that such destabilizing quadratic divergences are not important because whatever mechanism cancels the cosmological constant would presumably eliminate these terms, together with their potentially destabilizing implications. However, if the quadratic divergences could be arranged to cancel, one might hope that the cosmological constant problem could be addressed solely in terms of the low-energy effective theory. In this case, it might be possible to determine the weak scale physics by minimizing the low-energy effective potential.

Ref. [4] attempts to do precisely this by ensuring that all quadratic divergences cancel at one loop. These divergences are independent of the superpotential, so they can be arranged to cancel through scaling relations on the Kähler potential. The beauty of this scheme is that the largest contributions to the cosmological constant automatically vanish, independent of the details of the weak-scale physics (such as Yukawa couplings).

In this paper, we use our two-loop calculation to explicitly demonstrate that this situation does not persist at two loops. We find a quadratically divergent two-loop contribution to the vacuum energy which depends on the Yukawa couplings of the low-energy theory. It is difficult to envision a simple field theory mechanism, analogous to that proposed in [4], through which the quadratic divergence can be cancelled at two loops and beyond.

The outline of this paper is as follows. We first review the reasons for concern about quadratically divergent contributions to the effective potential. We identify the dangerous operators using the counting rules from Appendix A, and then focus on the problems associated with quadratically divergent contributions to tadpoles and the (field-dependent) vacuum energy.

In Section 3, we explain why the one-loop contributions to these quantities may vanish. We then show that the quadratically divergent contribution to the tadpole vanishes at one loop in supergravity theories with minimal Kähler potentials. (In Appendix B we extend this result to include higher-derivative nonrenormalizable operators in the Kähler potential.) We also discuss the vanishing of the one-loop vacuum energy in the LHC models, and explain why it is crucial for generating and maintaining the hierarchy in this class of theories [4].

In Section 4, we compute the quadratically divergent two-loop contribution to the effective potential. Our technique is explained in Appendix C. We use superspace Feynman rules and Kähler-Weyl invariance, which greatly simplifies the calculation and explicitly maintains the constraints of supersymmetry. We regulate the divergences with a simple momentum-space cutoff because it is sufficient to reveal the most important feature of the calculation, namely that there is indeed a quadratic divergence, but no term of the form $\Lambda^2 \log \Lambda$.

In Appendix D, we regularize the theory using higher-derivative operators. We find a different numerical coefficient for the quadratically divergent term, but the essential result remains the same: In theories where the quadratically divergent one-loop contribution vanishes, the two-loop contribution is the dominant divergence.

In the conclusion, we discuss the implications of our two-loop calculation, and speculate about higher loops. We argue that higher-loop quadratic divergences can destroy the hierarchy through quadratically divergent tadpoles, or by interfering with the mechanism that underlies the LHC models. The end result is that light gauge- and global-symmetry singlets are dangerous, and that LHC models require additional cancellations beyond one loop.

2   The Destabilizing Consequences of Quadratic Divergences

In renormalizable, globally supersymmetric theories, the naturalness of the hierarchy is guaranteed by the absence of quadratic divergences. When supersymmetry is softly broken, quadratic divergences do not appear at one loop and beyond.

In this paper we consider effective supersymmetric theories coupled to supergravity. These

theories necessarily contain supergravity-induced nonrenormalizable terms suppressed by powers of $1/M_P$, as well as other possible nonrenormalizable terms in the Kähler and superpotentials. We assume that supersymmetry is spontaneously broken, at a scale M_S, in a hidden sector which is coupled to the visible sector by interactions of gravitational strength. In the limit $M_P \to \infty$, we take the gravitino mass $M_{3/2} \simeq M_S^2/M_P$ to be held fixed, so the theory that describes the visible fields is globally supersymmetric with explicit soft supersymmetry-breaking terms.

As discussed above, supergravity is nonrenormalizable, so our theory must be considered as an effective theory, valid below a cutoff $\Lambda \simeq M_P$. Below the Planck scale, the lagrangian that describes the visible sector can contain any nonrenormalizable terms, consistent with the symmetries, with coefficients suppressed by appropriate powers of M_P. In general, after supersymmetry breaking, this leads to supersymmetry-breaking terms which are not soft. When inserted into quadratically divergent graphs, they may generate renormalizable terms whose coefficients scale with positive powers of Λ. Absorbing these terms into the low-energy parameters requires enormous fine-tuning and destabilizes the hierarchy in the visible sector.

In Appendix A, we argue that the dangerous diagrams are tadpole and vacuum graphs. These operators can have coefficients that scale with positive powers of M_P. In principle, mass terms can also be dangerous if they scale as a single power of the soft supersymmetry-breaking parameter $M_{3/2}$ and a single power of the cutoff. However, we show that such mass terms are relevant only in theories with a singlet where some field has a Planck-scale vev.

2.1   Tadpoles

In this section we show that quadratically divergent tadpoles will destabilize the hierarchy, provided the corresponding fields are light, and have renormalizable couplings to the other fields in the visible sector.[1] For simplicity, we will consider a supersymmetric theory in which the visible-sector fields include a vector-like Higgs representation, H_i $(i = 1, 2)$, and a gauge singlet, N. This toy model retains all the essential features for a discussion of destabilizing divergences.

We will first assume that the Kähler potential is minimal, in which case it splits into a sum of two pieces: one that involves the visible-sector fields, and the other their hidden-sector counterparts. This assumption is probably highly unnatural at the level of the nonrenormalizable terms in the Kähler potential. However, we will first explore the consequences of nonrenormalizable visible-sector

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[1]See also [5, 6]. For a discussion in the context of globally supersymmetric grand unification, see [7].

couplings, and only later generalize our analysis to the case of nonminimal, mixed-Kähler terms.

In what follows we assume a renormalizable superpotential,

$$W = \lambda N H_1 H_2 , \tag{2.1}$$

as well as a general visible-sector Kähler potential

$$K_{visible} = N^+ N + H^{i+} H_i + \frac{\alpha}{M_P} (N + N^+) H^{i+} H_i + \frac{\beta}{M_P} (\mathcal{D}^\alpha H_1 \, \mathcal{D}_\alpha H_2 + \text{h.c.}) + ... \tag{2.2}$$

In eq. (2.2), we include all terms consistent with symmetry,[2] up to order $1/M_P$.

We imagine that this theory is coupled to supergravity in the standard manner [8]. We also assume the existence of a hidden sector which is coupled to the observable sector by gravitational interactions. The hidden sector is assumed to spontaneously break supersymmetry at a scale

$$M_S^4 = \langle V_{hidden} \rangle = \langle K^{ij*} D_i W \, D_{j*} \bar{W} \rangle , \tag{2.3}$$

where the summation is over the hidden sector fields,

$$D_i W \equiv \partial_i W + \frac{1}{M_P^2} K_i W \tag{2.4}$$

is the Kähler-covariant derivative of the superpotential, and K^{ij*} is the inverse Kähler metric. The scale M_S is of order $10^{11} - 10^{14}$ GeV, depending on the details of the supersymmetry breaking.

As usual, spontaneous supersymmetry breaking in the hidden sector leads to explicit soft super-symmetry breaking in the visible sector. In components, one finds

$$V_{soft} = M_{3/2}^2 (n^* n + h^{i*} h_i) , \tag{2.5}$$

where n, h_i denote the scalar components of the superfields N, H_i. The Planck-suppressed terms in (2.2) induce hard supersymmetry-breaking operators, such as the nonholomorphic trilinear scalar interactions

$$V_{hard} = \frac{M_{3/2}^2}{M_P} (n^* + n) \, h^{i*} h_i . \tag{2.6}$$

It is important to note that there is a definite relation between the coefficients in eqs. (2.2), (2.5) and (2.6). This relation is important for the cancellation at the one-loop order (see Sect. 3); it does not persist to higher loops.

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[2]For simplicity we omit any vector superfields because they are inessential in our analysis.

Terms of the type (2.6) are known to introduce quadratic divergences [1], and can potentially destabilize the hierarchy if they induce a quadratically divergent tadpole. To see this, consider the scalar component n of N. Let us assume that n has mass M_N. This may be a soft mass, in which case it is of order $M_{3/2}$, or an arbitrary supersymmetric mass from an $M_N N^2$ term in the superpotential. If a tadpole is generated, the scalar potential for n becomes

$$M_{3/2}^2 \frac{\Lambda^2}{M_P} (n + n^*) + M_N^2 n^* n .$$

(2.7)

The vev of n shifts by

$$\langle \delta n \rangle \simeq \frac{\Lambda^2}{M_P} \left(\frac{M_{3/2}}{M_N} \right)^2 ,$$

(2.8)

in which case the fields H_i acquire a mass of the order

$$\mu_{12} \simeq \frac{\Lambda^2}{M_P} \left(\frac{M_{3/2}}{M_N} \right)^2 .$$

(2.9)

For the hierarchy to be stable, we must require $\mu_{12} \simeq M_{3/2}$. In this case, eq. (2.9) implies that the cutoff Λ must be less than $\sqrt{M_N^2 M_P / M_{3/2}}$. Naturalness places an upper bound on the scale of new physics in hidden-sector theories with visible singlets.[3]

For the case of a light singlet, with $M_N \simeq M_{3/2}$, the upper bound becomes $\Lambda \lesssim \sqrt{M_{3/2} M_P}$, which is precisely the scale of supersymmetry breaking in the hidden sector. If we turn this around, and let the cutoff be M_P, we find that the singlet must be heavier than the intermediate scale, $M_N \gtrsim \sqrt{M_{3/2} M_P}$.

Thus we have seen that the hierarchy is destabilized in the presence of gauge- and global-symmetry singlets with renormalizable visible-sector interactions – *provided* that a quadratically divergent tadpole is generated. In Sect. 4 we will see that such tadpoles do not always appear at one loop, but are, in fact, induced at two-loop order.

2.2 Vacuum Energy

Quadratically divergent contributions to the vacuum energy represent a serious unsolved problem in relation to the cosmological constant. They also destabilize the hierarchy in effective supergravity theories with a sliding gravitino mass [4]. In these models, the gravitino mass either turns out to

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[3]As will be evident from sect. 4, it is also possible to induce a direct $h_1 h_2$-mass term if there are vevs of order M_P in the hidden sector.

be too large (of order the Planck mass) or too small (zero). The stability of the hierarchy requires that the cosmological constant vanish to order $M_{3/2}^4$.

The authors of Ref. [4] enumerated a set of conditions under which the vacuum diagrams cancel to one loop. They found that the cancellation requires a relation between the hidden- and visible-sector contributions to the one-loop effective potential. Furthermore, they proposed that the cancellation does not depend on the parameters of the superpotential and can be understood in terms of a geometric property of the Kähler potential.

If this situation were to persist at higher-loop order, it would allow the effective superpotential to be determined dynamically, as in Ref. [4]. If, however, the higher-order visible-sector contributions introduce new quadratic divergences that *depend on the superpotential*, the simplicity of this mechanism would be called into question. The point is that in this case, the cancellation would require the hidden sector to depend on the same Yukawa couplings as the visible sector. Furthermore, the hidden-sector fields would have to be sensitive to the same radiative corrections as the visible-sector fields. Such a situation would be unprecedented and would probably require a miracle of string theory. Moreover, the fact that the visible and hidden sectors depend on the same Yukawa couplings also calls into question the calculability of these theories.

For this reason, we believe that it is important to calculate the effective potential to two-loop order. In Sect. 4 we will find a Yukawa-dependent, two-loop, quadratically divergent contribution to the vacuum energy.[4] This contribution will destabilize the hierarchy in the LHC models.

3 The Dangerous Graphs at One Loop

3.1 Tadpoles

In Ref. [3], it was shown that no quadratically divergent tadpoles are induced in hidden-sector models with minimal Kähler potentials. This conclusion is based on Refs. [10, 11], in which the divergent parts are calculated for the bosonic contribution to the one-loop supergravity effective action. This result is specific to spontaneously broken supergravity and, as shown below, relies on a cancellation between the mass and wave function renormalizations. In this section we will confirm this result in terms of our toy model. In the sect. 4 we shall see that the cancellation is an accident of the one-loop approximation.

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[4]This contribution to the vacuum energy was also discussed in [9].

In what follows we will ignore the higher-derivative term[5] in (2.2). Therefore the kinetic terms for the matter fields are simply [8]

$$K_{ij*} \partial_\mu z^i \partial^\mu z^{j*} \; + \; i K_{ij*} \bar{\chi}^i \sigma^\mu \partial_\mu \chi^{j*} \; . \tag{3.1}$$

In this expression, the Kähler metric depends on the coefficient α through field-dependent terms.

In a similar way, the scalar potential is just

$$V \; = \; \exp(K/M_P^2) \; \left(K^{ij*} \; D_i W \; D_{j*} \bar{W} \; - \; \frac{3 W \bar{W}}{M_P^2} \right) \; . \tag{3.2}$$

Here $K = K_{visible} + K_{hidden}$, $W = W_{visible} + W_{hidden}$. For our toy model, we cancel the cosmological constant by adjusting a constant term

$$W_0 = M_S^2 M_P / \sqrt{3} \tag{3.3}$$

in the superpotential.

The constant W_0 introduces soft supersymmetry-breaking terms in the scalar potential of the observable sector. These terms can be found from (2.2) and (3.2). Expanding in inverse powers of M_P, we find the following scalar potential for the visible fields:[6]

$$V_{obs} \; = \; K^{ij*} \; W_i \; \bar{W}_{j*} \; + \; M_{3/2}^2 \; K^{ij*} \; K_i \; K_{j*} \; + \; \mathcal{O}\left(\frac{M_{3/2}^2}{M_P^2}\right) \; . \tag{3.4}$$

Here K is the Kähler potential (2.2) for the observable fields, and $M_{3/2}$ is the gravitino mass, $M_{3/2} = M_S^2 / \sqrt{3} M_P$. The first term in (3.4) is the usual supersymmetric scalar potential, while the second contains the terms that break supersymmetry.

In terms of component fields, it is not hard to see that the following terms have the potential to contribute to a quadratically divergent one-loop tadpole for the singlet field n:

$$\alpha \; \frac{M_{3/2}^2}{M_P} \; (n + n^*) \; h^{i*} h_i \; , \tag{3.5}$$

from (3.4), (2.6), and

$$\frac{\alpha}{M_P} \; n \; \partial_\mu h^{i*} \partial^\mu h_i \; , \tag{3.6}$$

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[5]In appendix B, we show that the higher-derivative term with coefficient β is equivalent to the trilinear term with coefficient α, to order $1/M_P$.

[6]We have assumed that there are no terms in K which are bilinear both in the hidden and observable fields. Such terms contribute to nonuniversal soft scalar masses.

which follows from (3.1), (2.2). These terms give rise to two quadratically divergent one-loop graphs: one with the supersymmetric term (3.6) at the vertex and a soft supersymmetry-breaking insertion (2.5) in the propagator, and another with the supersymmetry-breaking vertex (3.5) and no insertions in the propagator. The relative sign and magnitude of these two contributions are such that the quadratic divergence cancels.

This confirms Jain's result from Ref. [3], in which he pointed out that the kinetic energy and potential renormalizations are such that all quadratically divergent tadpoles cancel after rescaling the fields to their conventional normalization. The cancellation in terms of Feynman graphs represents the same physics.

As this example illustrates, the one-loop cancellation of the singlet tadpole follows from the assumption of a minimal Kähler potential in the supergravity lagrangian. The cancellation relies on the fact that the proportionality constant between the soft supersymmetry-breaking mass term and the quadratic term in the Kähler potential is precisely the same as that between the supersymmetry-violating trilinear scalar vertex and the trilinear term in the Kähler potential. Should there exist nonrenormalizable terms in the Kähler potential that mix the hidden- and visible-sector fields, such as $\Phi^+\Phi H^+HN/M_P^3$ or $\Phi^+\Phi H^+H/M_P^2$, where Φ is a hidden-sector field with nonzero F_Φ, then there would be additional contributions to the soft supersymmetry breaking that can destroy the proportionality relations. A similar conclusion was reached in [3].

This one-loop cancellation can also be understood from a different point of view. After the superfield redefinition

$$H_i \ \rightarrow \ H_i \ \left(1 \ - \ \alpha \ \frac{N}{M_P}\right) \ , \tag{3.7}$$

the Kähler potential (2.2) and superpotential (2.1) become (with $\beta = 0$)

$$K \ = \ N^+N \ + \ H^{i+}H_i \ + \ \mathcal{O}\left(\frac{1}{M_P^2}\right) \ , \tag{3.8}$$

$$W \ = \ \lambda N H_1 H_2 \ - \ \lambda \, \alpha \, \frac{N^2}{M_P} \, H_1 H_2 \ + \ \mathcal{O}\left(\frac{1}{M_P^2}\right) \ . \tag{3.9}$$

After the field redefinition, there are no interactions that could possibly generate a one-loop quadratically divergent tadpole.[7] This field redefinition eliminates both component terms (3.5, 3.6) that contribute to the one-loop tadpole. The field redefinition also simplifies the two-loop calculation in sect. 4.

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[7]This result is reminiscent of the Ademollo-Gatto result, in which a symmetry is explicitly broken, and the leading correction can be defined away, but a symmetry-breaking effect appears at next-to-leading order.

3.2 Vacuum Energy

In this section, we will discuss the one-loop contribution to the (field-dependent) vacuum energy. As explained previously, we will work in superspace, and exploit the fact that all divergent graphs assemble into contributions to the Kähler potential. When necessary, we will introduce supersymmetry breaking by inserting F-type vevs into our superspace expressions.

In rigid supersymmetry, the one-loop quadratically divergent contribution to the Kähler potential is known to be

$$\frac{\Lambda^2}{16\pi^2} \int d^4\theta \, \log \det K_{ij*} \, . \tag{3.10}$$

Supergravity introduces an additional contribution, proportional to

$$\frac{\Lambda^2}{16\pi^2} \int d^4\theta \, \frac{K}{M_P^2} \, . \tag{3.11}$$

In this paper, we will not attempt to calculate the precise coefficient of the supergravity term; this has been done in [12]. Note that the supergravity term does not involve the superpotential.

Both of these terms contribute to the one-loop quadratically divergent vacuum energy. After inserting supersymmetry-breaking F-term vevs, one finds contributions of the form

$$\frac{\Lambda^2}{16\pi^2} \int d^4\theta \, \log \det K_{ij*} \quad = \quad \frac{\Lambda^2}{16\pi^2} \, (\log \det K_{ij*})_{\ell m*} F^\ell \, F^{*m} \, ,$$

$$\frac{\Lambda^2}{16\pi^2} \int d^4\theta \, \frac{K}{M_P^2} \quad = \quad \frac{\Lambda^2}{16\pi^2} \frac{K_{\ell m*} F^\ell F^{*m}}{M_P^2} \, . \tag{3.12}$$

These terms collect themselves into the one-loop effective potential,

$$V_{1-loop} \;=\; \frac{1}{32\pi^2} \, \Lambda^2 \, \mathrm{Str}\left[\mathcal{M}(M_{3/2})\right]^2 \;-\; \frac{1}{64\pi^2} \, \mathrm{Str}\left[\mathcal{M}(M_{3/2})\right]^4 \log \frac{\Lambda^2}{\left[\mathcal{M}(M_{3/2})\right]^2} \, , \tag{3.13}$$

where $\mathcal{M}(M_{3/2})$ is the gravitino-mass-dependent mass matrix.

In a usual field theory calculation, the cutoff dependence would be absorbed by Λ-dependent counterterms. In a calculation from string theory, the sum of the heavy and light modes should turn the cutoff into a physical scale related to the Planck scale. In either case, if the $\mathrm{Str}\mathcal{M}^2$ does not vanish, there is a quadratically divergent contribution to the effective potential.

In models with a sliding gravitino mass, the scale of supersymmetry breaking is determined by the one-loop effective potential. If $\mathrm{Str}\mathcal{M}^2 \neq 0$, the natural scale for the gravitino mass is M_P or

zero. If, however, the Kähler potential is such that $\mathrm{Str}\mathcal{M}^2 = 0$, then the gravitino mass may be stabilized at an exponentially smaller scale. This is the hope that underlies the LHC mechanism.

The LHC models, however, rely heavily on the precise form of the quadratic divergences. It is natural to ask whether the one-loop cancellation persists to higher-loop order. We address this question in the following section.

4   The Dangerous Graphs at Two Loops

In the previous section, we saw that dangerous quadratically divergent tadpoles vanish at one loop in theories with minimal Kähler potentials. We also saw that the quadratically divergent vacuum energy vanishes in a class of supergravity models, the LHC models. In each case, the question of naturalness requires a two-loop analysis. Therefore in this section, we compute the quadratically divergent field-dependent effective potential at two loops. Our result can be used to find the quadratically divergent tadpole and vacuum energy.

An important difference between one and two loops is that the one-loop quadratically divergent effective potential depends only on the Kähler potential. At one loop, the vacuum energy is independent of the superpotential of the theory. Diagramatically, however, it is clear that the vacuum energy depends on the Yukawa couplings at higher-loop order. In fact, the two-loop quadratic divergence is due, in part, to the fact that the one-loop Kähler potential depends on the Yukawa couplings. These Yukawa terms violate the proportionalities among the soft supersymmetry-breaking terms that are necessary for the cancellation of the tadpole. They also violate the scaling properties of the Kähler potential that were imposed for successful implementation of the LHC mechanism. (Of course, the full two-loop answer is not obtained by substituting one-loop results into the Kähler potential, but requires a true two-loop calculation, which we do here.)

As we already mentioned, we choose to do our calculations in superspace. This is the simplest way (and in practice, the only way) to calculate quadratic divergences and ensure that the supersymmetric constraints are maintained. It is sufficient to calculate in the supersymmetric theory because all quadratically divergent graphs, including those that involve soft supersymmetry breaking, are related to manifestly supersymmetric graphs by the insertion of supersymmetry-breaking vevs. As we will see, the full power of this procedure is realized when one also enforces the constraints from super-Weyl-Kähler invariance, which imply definite relations between the supersymmetric and supersymmetry-breaking contributions to the effective potential.

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In preparation for our two-loop discussion, we will first describe the calculation of the Yukawa-dependent, one-loop logarithmically divergent contribution to the effective potential. In the globally supersymmetric limit, the only logarithmically divergent graph is shown in Fig. 1. The calculation is trivial in superspace; it yields the operator

$$\frac{\log\Lambda^2}{32\pi^2} \int d^4\theta \; W_{ij}\bar{W}^{ij} \; . \tag{4.1}$$

When supergravity is included, however, there is much more to the story. There are important supergravity corrections to (4.1), which can most easily be found by performing a background-field calculation in superspace.

In a background field calculation, there appear two types of infinite counterterms – those that are invariant under background field reparametrization (on-shell counterterms) and those that are not invariant, but vanish on-shell (off-shell counterterms). The latter can be eliminated by field redefinitions and do not correspond to divergences of the S-matrix. Therefore we are only interested in the on-shell divergences of the theory.[8]

In rigid supersymmetry, the expression (4.1) is automatically invariant under redefinitions of the background fields because the superpotential transforms as a scalar under field redefinitions. In supergravity, however, the superpotential is not an ordinary holomorphic function of the chiral superfields, but is instead a section of a holomorphic line bundle [14]. This means that under field reparametrizations, both the superpotential and Kähler potential transform under Kähler transformations

$$W \quad \rightarrow \quad e^{-F/M_P^2} \; W \tag{4.2}$$

$$K \quad \rightarrow \quad K \; + \; F \; + \; \bar{F} \; , \tag{4.3}$$

where F ($\bar{F}$) is a holomorphic (antiholomorphic) function of the complex fields.

In the superspace formulation of supergravity, the functions F must be promoted to holomorphic functions of the chiral superfields. In addition, the Kähler transformation (4.2) must be accompanied by a super-Weyl rescaling of the vielbein (see [8], Appendix C, and eq. (4.6) below). The resulting super-Weyl-Kähler transformations impose important constraints on the superspace theory. They ensure that the component action is Kähler invariant after eliminating the auxiliary fields and rescaling the metric to impose a canonical normalization on the Einstein action.

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[8]For a superspace discussion of the $2d$ supersymmetric sigma model, see Ref. [13]. A related component discussion in $4d$ supergravity is given in Ref. [10].

The on-shell divergent counterterms in spontaneously broken supergravity must respect the symmetries of the underlying theory and therefore be super-Weyl-Kähler invariant. The invariant counterterm can be obtained from (4.1) by inserting a factor of $e^{2K/3M_P^2}\varphi\bar{\varphi}$ in the integrand. One finds[9]

$$\frac{\log\Lambda^2}{32\pi^2}\int d^4\theta \; e^{2K/3M_P^2} \; \varphi\bar{\varphi} \; W_{ij}\bar{W}^{ij} \; , \tag{4.4}$$

where φ is the conformal compensator superfield of supergravity theory. In addition, the derivatives must be Kähler-covariant, which implies

$$\begin{aligned}
D_i &\equiv& e^{K/M_P^2} \; \partial_i \; e^{-K/M_P^2} \\
W_i &\equiv& D_i W \\
W_{ij} &\equiv& D_j W_i \; - \; \Gamma_{ij}^k W_k \; ,
\end{aligned} \tag{4.5}$$

where Γ_{ij}^k is the connection of the Kähler manifold.

The chiral compensator φ transforms as follows under super-Weyl-Kähler transformations,

$$\begin{aligned}
\varphi &\rightarrow& e^{F/3M_P^2} \; \varphi \\
\bar{\varphi} &\rightarrow& e^{\bar{F}/3M_P^2} \; \bar{\varphi} \; .
\end{aligned} \tag{4.6}$$

With these transformations, it is easy to see that the term (4.4) is super-Weyl-Kähler invariant.

This one-loop term in the effective Kähler potential has important physical consequences. The rigid piece of (4.4) determines the anomalous dimensions of the chiral superfields. The vevs of the $e^{2K/3M^2}$ factor and of the chiral compensators play the roles of spurions, so (see Appendix C)

$$\begin{aligned}
e^{2K/3M_P^2} &=& e^{2K/3M_P^2} \left[1 \; + \; \theta^2 \frac{2}{3}K_i F^i \; + \; \bar{\theta}^2 \frac{2}{3}K_{i*}F^{*i} \right. \\
&& \left. + \; \theta^2\bar{\theta}^2 \frac{2}{3}\left(K_{ij*} \; + \; \frac{2}{3}K_i K_{j*}\right) \; F^i F^{*j} \right] \\
\varphi &=& e^{K/6M_P^2}\left[1 + \theta^2\left(e^{K/2M_P^2}\frac{\bar{W}}{M_P^2} \; + \; \frac{K_i F^i}{3M_P^2}\right)\right] \\
\bar{\varphi} &=& e^{K/6M_P^2}\left[1 + \bar{\theta}^2\left(e^{K/2M_P^2}\frac{W}{M_P^2} \; + \; \frac{K_{i*}F^{*i}}{3M_P^2}\right)\right] \; ,
\end{aligned} \tag{4.7}$$

where terms on the l.h.s. should be interpreted as superfield vevs, while the terms on the r.h.s. are functions of the vevs of the scalar components of the chiral superfields. Inserting these expressions

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[9]Details of the derivation of (4.4) are given in Appendix C.

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into the logarithmically divergent counterterm (4.4), we find the logarithmically divergent terms that are responsible for the running of the soft scalar masses,

$$\frac{\log\Lambda^2}{32\pi^2} \int d^4\theta \; e^{2K/3M_P^2} \; \varphi\bar{\varphi} \; W_{ij}\bar{W}^{ij} \; \supset \; \frac{\log\Lambda^2}{32\pi^2} \; \frac{M_S^4}{M_P^2} \; \lambda_{ijk}\bar{\lambda}^{ij}{}_\ell \; \phi^k\phi^{*\ell} \; . \tag{4.8}$$

Here the λ_{ijk} are the Yukawa couplings and ϕ^k are the scalar components of the visible-sector fields. To derive this expression, we also used the facts that

$$K_{\ell m*}F^\ell F^{*m} \;=\; M_S^4 \tag{4.9}$$

and

$$\frac{W\bar{W}}{M_P^4} \;=\; \frac{M_S^4}{3M_P^2} \; . \tag{4.10}$$

From the expression (4.8), one can immediately find the Yukawa-coupling-dependent part of the one-loop beta functions for the soft masses. Note that one would have found the same answer for the soft masses by inserting e^K instead of $\varphi\bar{\varphi}e^{2K/3}$. However, this procedure does *not* give the correct answer for the soft supersymmetry-breaking trilinear terms. This illustrates the importance of the compensator formalism in superspace supergravity theories.

This example demonstrates that the leading (in terms of $1/M_P$) divergent contributions to the visible-sector effective potential can be found by the following procedure:

1. Calculate the divergent part of a rigid supergraph.

2. Cast the resulting operator in a form that is invariant under supergravity field redefinitions. For graphs with only chiral vertices and no purely chiral propagators, this means multiplying the integrand by a factor of

$$\left(e^{K/3M_P^2}\right)^P \; (\varphi\bar{\varphi})^{3V-P} \; , \tag{4.11}$$

   where P is the number of $\Phi^+\Phi$ propagators and V the number of chiral vertices.

3. Insert supersymmetry-breaking vevs for the chiral compensator and the hidden-sector fields.

The resulting component expressions give the divergent contributions to the supersymmetric and soft supersymmetry-breaking operators. For the case at hand, the result (4.4) coincides exactly with the one obtained by Jain and Gaillard [10] from a component calculation of the one-loop effective action in an arbitrary bosonic background.

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Another effect of supergravity is to induce additional terms that are suppressed by more powers of $1/M_P$. These terms are induced by diagrams with gravitational fields in the loops. We will not compute such terms in this paper; at one loop, the logarithmically divergent terms were found in Ref. [10]:

$$\delta K = -\frac{\log \Lambda^2}{32\pi^2} \; e^{K/M_P^2} \left(2 \frac{W_i \bar{W}^i}{M_P^2} + 4 \frac{W \bar{W}}{M_P^4} \right) \; . \tag{4.12}$$

These terms are exactly what one would expect by naive power counting. Their effects on the renormalization of the low-energy theory are suppressed by powers of $1/M_P$.

We shall follow the same general procedure in our search for two-loop quadratically divergent operators. We will first look for a quadratically divergent graph in rigid supersymmetry, and then dress it according to the background field prescription. In this way we will find leading two-loop contributions to the effective potential.

Motivated by LHC models, we will seek a quadratically divergent graph that depends on the visible-sector superpotential. (There are other graphs that depend only on the Kähler potential.) Such a quadratically divergent supergraph is shown in Fig. 2. After applying the standard super-Feynman rules, we obtain the following expression for the operator generated by this graph:

$$\frac{1}{3!} \int d^4\theta \; W_{ijk} \bar{W}^{ijk} \; \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} \frac{1}{k^2 q^2 (k+q)^2}. \tag{4.13}$$

For simplicity, we calculate the integral with a hard momentum-space cutoff.

The integral in (4.13) is clearly quadratically divergent. Rewriting it as

$$\int\limits_{|k|,|p|<\Lambda} \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} \frac{1}{k^2 q^2 (k+q)^2} = \frac{\Lambda^2}{512\pi^4} \int\limits_0^1 dx \int\limits_0^1 dy \; \frac{x+y-|x-y|}{xy} \; , \tag{4.14}$$

where the term on the r.h.s. follows from performing the angular integration, we see that there are no infrared divergences. (Note that the infrared and ultraviolet divergences would be related, since on dimensional grounds they would appear as $\Lambda^2 \log(\Lambda/m)$.) The absence of the logarithm justifies the local operator in (4.13). After performing the integral, we find

$$\frac{1}{3!} \frac{\Lambda^2}{128\pi^4} \int d^4\theta \; W_{ijk} \bar{W}^{ijk} \; . \tag{4.15}$$

This calculation, with a hard cutoff, shows that there is indeed a quadratically divergent contribution at two loops. It also shows that there are no infrared logarithms associated with this graph.

This calculation might be criticized because a hard momentum-space cutoff violates supersymmetry. In Appendix D, we repeat the calculation with a higher-derivative regulator [15], and still find that the operator (4.15) is induced. This regulator preserves supersymmetry provided that the higher derivatives are covariantized appropriately. In Ref. [11], Pauli-Villars regularization was used to calculate the quadratically divergent one-loop graphs. This works only to one loop [15]; at higher orders it must be modified by some other scheme, such as higher covariant derivatives.

In the locally supersymmetric case, the operator (4.15) can be written in the same super-Weyl-Kähler and field-redefinition-invariant form as (4.4):

$$\frac{1}{3!} \frac{\Lambda^2}{(16\pi^2)^2} \int d^4\theta \; e^{K/M_P^2} \; W_{ijk}\bar{W}^{ijk} \; . \tag{4.16}$$

Note the absence of a compensator contribution, which follows from the Feynman rules explained in Appendix C.

The operator (4.16) has important consequences for the mass hierarchy. If we substitute the observable superpotential of our toy model (3.9) into (4.16), we generate a term of the form

$$\frac{\Lambda^2}{(16\pi^2)^2} \; 2\lambda^2\alpha \int d^4\theta \; e^{K/M_P^2} \; \frac{N + N^\dagger}{M_P} \; . \tag{4.17}$$

This term induces a quadratically divergent tadpole for the scalar component n of N, as can be seen by inserting the hidden-sector F-term vevs into the Kähler potential. This tadpole clearly destabilizes the hierarchy.[10] The operator (4.16) also induces a field-dependent quadratically divergent contribution to the vacuum energy:

$$V \;\; = \;\; -\frac{1}{3!} \frac{\Lambda^2}{(16\pi^2)^2} \int d^4\theta \; e^{K/M_P^2} \; W_{ijk}\bar{W}^{ijk} \;\; \simeq \;\; -\frac{\Lambda^2}{(16\pi^2)^2} \; \frac{M_S^4}{M_P^2} \; \frac{1}{3!} \; |\lambda_{ijk}|^2 \;\; + \;\; ... \; , \tag{4.18}$$

where the λ_{ijk} are the Yukawa couplings in the theory. As discussed previously, this term will destabilize the hierarchy of the LHC models.

5   Implications and Conclusions

In this paper we have studied the question of naturalness in effective supersymmetric theories. In particular, we studied the potentially destabilizing quadratic divergences that are induced at one- and two-loop order in the effective potential.

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[10]Note (4.17) also implies that if there are M_P vevs in the hidden sector, there is also a direct contribution of order $M_{3/2}M_P$ to the Higgs mass term from the F-component of the singlet, $F_N \sim h_1 h_2$.

We took a careful look at the generation of divergent tadpole diagrams. We confirmed the result of Jain, that gauge- and global-symmetry singlets do *not* develop tadpoles at one-loop order, provided that the Kähler potential is minimal. At two loops, we showed that a quadratically divergent tadpole can indeed be generated.

Our results indicate that the one-loop result is an accident of the one-loop approximation. This conclusion is in accord with our notions of naturalness because there is no symmetry that would forbid a divergent tadpole. Since there is no symmetry, it has to appear, and indeed it does.

We also took a second look at the so-called LHC models of Ferrara, Kounnas and Zwirner. These models rely on a cancellation of the cosmological constant to order $M_{3/2}^4$. At one loop such a cancellation can be enforced by choosing a special Kähler potential for the visible and invisible sectors. Again, since this is not related to a symmetry of the theory, we expect a contribution to arise at higher loops. Indeed, at two loops we found that the cancellation is spoiled by terms that depend on the superpotential of the visible sector. Our results imply that LHC models do not work unless there is some conspiracy between the visible and invisible worlds which cancels the Yukawa-dependent divergence. Such a cancellation would be difficult to understand at the level of effective field theory.

Our calculations can be readily generalized to higher loops. The superpotential-dependent divergences can be guessed by induction. At one loop, we found logarithmically divergent contributions to the component Kähler potential which go like

$$\log(\Lambda^2) \, e^{K/M_P^2} \, (W_{ij}\bar{W}^{ij} \; + \; \frac{1}{M_P^2}W_i\bar{W}^i \; + \; \frac{1}{M_P^4}W\bar{W}) \, . \tag{5.1}$$

At two loops, we found quadratically divergent terms such as

$$\Lambda^2 \, e^{K/M_P^2} \, (W_{ijk}\bar{W}^{ijk} \; + \; \frac{1}{M_P^2}W_{ij}\bar{W}^{ij} \; + \; \frac{1}{M_P^4}W_i\bar{W}^i \; + \; \frac{1}{M_P^6}W\bar{W}) \, . \tag{5.2}$$

Therefore at three loops, we expect quartically divergent terms of the form

$$\Lambda^4 \, e^{K/M_P^2} \, (W_{ijk\ell}\bar{W}^{ijk\ell} \; + \; \frac{1}{M_P^2}W_{ijk}\bar{W}^{ijk} \; + \; \frac{1}{M_P^4}W_{ij}\bar{W}^{ij} \; + \; \frac{1}{M_P^6}W_i\bar{W}^i \; + \; \frac{1}{M_P^8}W\bar{W}) \, . \tag{5.3}$$

The leading term comes from a rigid supersymmetry graph, while the other terms come from graphs with supergravity fields in the loops.

Taking $\Lambda \simeq M_P$, we see that the three-loop terms induce new possibilities for destabilizing divergences. For example, the $W_{ijk\ell}\bar{W}^{ijk\ell}$ term also contains a quadratically divergent tadpole. Note

that this means we expect new superpotential-dependent divergences at all orders of perturbation theory. Moreover, it implies that the cancellation of quadratic divergences would require a conspiracy between terms at all orders in the loop expansion. Clearly, if LHC models are to work, there must be a deep reason to explain this miracle. Presumably, this is related to the cosmological constant problem, about which (once again) we have nothing to say.

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A Superspace Counting Rules

In an ordinary field theory, operators of dimension three or less can have coefficients that scale with a positive power of the cutoff. In a spontaneously broken supersymmetric theory, supersymmetric cancellations imply that at least one power of the cutoff is replaced by $M_{3/2}$, the scale of supersymmetry breaking in the visible sector.

The supersymmetric cancellations are such that quadratically divergent diagrams affect the vacuum energy. Such divergences can destabilize the hierarchy in supergravity theories with a sliding gravitino mass [4]. In addition, tadpoles can have quadratically divergent coefficients in theories with singlets. Finally, hidden-sector supersymmetric theories have potentially dangerous mass renormalizations, of order $M_{3/2}M_P \simeq M_S^2$. However, divergent mass terms, of order $M_{3/2}M_P$, can also only be generated in theories with singlets.

The divergence structure of spontaneously broken supersymmetric theories can be obtained from a manifestly supersymmetric calculation for which the power counting can be done in superspace. Superspace power counting automatically incorporates supersymmetric cancellations, and provides an efficient way to identify dangerous graphs. Supersymmetry breaking can be accommodated by inserting supersymmetry-breaking vevs in the supersymmetric operators induced by the dangerous diagrams.

The usual formula for the superficial degree of divergence of a D-type superspace diagram can be readily generalized to include nonrenormalizable operators of the type present in (2.2). It becomes

$$D \;\leq\; 2 \;-\; E_c \;-\; P_c \;+\; \sum_d dV_d \;, \tag{A.1}$$

where E_c denotes the number of chiral external legs, P_c represents the number of chiral $\langle\Phi\Phi\rangle$ propagators, and V_d denotes the number of nonrenormalizable operators, suppressed by $(1/M_P)^d$. Using this formula, it is easy to see that a given diagram is proportional to

$$\Lambda^D \prod_d \left(\frac{1}{M_P}\right)^{dV_d} . \tag{A.2}$$

If we take the cutoff Λ to be of order M_P, this reduces to

$$D \;\leq\; M_P^{2-E_c-P_c} \;. \tag{A.3}$$

Equation (A.3) implies that a vacuum contribution can be proportional to M_P^2, while a superspace tadpole diagram can be at most linearly divergent, that is, proportional to M_P. In a similar

way, the divergence associated with a superspace two-point function can be at most logarithmic, provided there are no Planck-scale vevs. Therefore models without singlet superfields are automatically safe from divergent tadpoles. In particular, the minimal supersymmetric standard model is safe from destabilizing tadpoles [16].

B   Higher-Derivative Terms and the One-Loop Tadpole

In this Appendix we will study higher-derivative terms in the Kähler potential. To order $1/M_P$, we will find that they are equivalent to trilinear terms, so they do not induce one-loop quadratically divergent tadpoles.

To be concrete, let us consider the same toy model as in the text, specified by (2.1), (2.2), including the term with the higher supercovariant derivatives. After supersymmetry breaking, this term leads to a Dirac mass for the fermionic components χ_i of the superfields H_i,

$$\beta \, \frac{M_{3/2}^2}{M_P} \, \chi_1 \chi_2 \; . \tag{B.1}$$

The mass (B.1) is hard, and when inserted into a one-loop tadpole, it can lead to an uncanceled quadratic divergence [1]. The calculation of Ref. [10] did not include such higher-derivative terms in the tree-level supergravity Lagrangian.

We shall start our analysis by integrating by parts in the supergravity Lagrangian,[11]

$$\mathcal{D}^\alpha H_1 \, \mathcal{D}_\alpha H_2 \;\; \rightarrow \;\; - \, H_1 \, \mathcal{D}^\alpha \mathcal{D}_\alpha H_2 \; . \tag{B.2}$$

We then perform the field redefinition

$$
\begin{aligned}
H_1 &\;\; \rightarrow \;\; H_1 \; + \; \frac{\beta}{M_P} \, (\bar{\mathcal{D}}^2 - 8R) H_2^+ \\
H_2 &\;\; \rightarrow \;\; H_2 \; .
\end{aligned}
\tag{B.3}
$$

This field redefinition is manifestly supersymmetric since $(\bar{\mathcal{D}}^2 - 8R)$ is a chiral projector [8, 17].

After the field redefinition, the Kähler potential is simply

$$K_{visible} \; = \; N^+ N \; + \; H^{i+} H_i \; + \; \alpha \, \frac{N + N^+}{M_P} \, H^{i+} H_i \; + \; \mathcal{O}\left(\frac{1}{M_P^2}\right) \; . \tag{B.4}$$

In this expression, we have neglected terms proportional to $R/M_P \simeq M_{3/2}/M_P$, since after super-symmetry breaking, the vev of the superfield R is of order $M_{3/2}$ [8], and fluctuations around the vev are suppressed by additional powers of M_P.

Similarly, the superpotential becomes

$$W \; = \; \lambda \, N H_1 H_2 \; + \; \frac{\lambda \beta}{M_P} \, N(\bar{\mathcal{D}}^2 - 8R) H_2^+ H_2 \; . \tag{B.5}$$

---

[11] We use the identity [17] $0 = \int d^4x d^4\theta \, \partial_M (E v^A E_A^M)(-1)^a = \int d^4x d^4\theta \, E \mathcal{D}_A v^A (-1)^a$.

The second term in W is actually a term in the Kähler potential [17],

$$\int d^2\theta\, \mathcal{E} N(\bar{\mathcal{D}}^2 - 8R)H_2^+ H_2 \;=\; \int d^2\theta\, \mathcal{E}(\bar{\mathcal{D}}^2 - 8R)NH_2^+ H_2 \;=\; -4\int d^4\theta\, E N H_2^+ H_2\,. \qquad \text{(B.6)}$$

This shows that the higher-derivative term $\mathcal{D}^\alpha H_1 \mathcal{D}_\alpha H_2$ in the Kähler potential (2.2) is equivalent to the trilinear $NH_2^+ H_2$, to order $1/M_P$,

Thus we can conclude that at the one-loop level, the higher-derivative term does not induce a quadratically divergent tadpole, in contrast with the naive expectation [2]. (We have also verified the cancellation by explicitly calculating the one-loop component graphs that are induced by this term.)

C   Superspace Loop Calculations in Spontaneously Broken Supergravity

As was already mentioned in sect. 2, we assume supergravity to be a low-energy effective theory, valid below some cutoff scale $\Lambda \lesssim M_P$. We are interested in the predictions of the theory for the observable-sector interactions. We assume the validity of a perturbative loop expansion, under the assumption that visible-sector fields are defined such that their vevs are less than M_P. (If there were vevs of order M_P, the effective propagator would have to be generalized to incorporate higher-order quadratic terms.)

For the purposes of our calculations, we restrict our attention to the leading diagrams that involve Yukawa couplings. (Recall that the classical Kähler metric does not depend on the superpotential.) We work to two-loop order, so we do not need to consider contributions from gauge, gravity or hidden-sector loops.[12] In fact, because we assume F-type supersymmetry breaking, we can neglect gauge interactions altogether. Furthermore, the hidden-sector and gravitational fields contribute to the visible-sector operators only through their vevs.

In this section we present a very powerful method for calculating the divergent loop contributions to soft supersymmetry-breaking operators, under these assumptions. We show how to derive the divergent part of the soft supersymmetry-breaking effective potential in supergravity from a calculation in the globally supersymmetric theory. Our methods permit us to use a regulator appropriate to the supersymmetric theory, and furthermore to exploit the power of the superspace formalism.

Therefore at scales below the cutoff, we consider the visible fields, plus the gravitational and hidden-sector fields which acquire vevs as a result of supersymmetry breaking. The gravitational fields are contained in an $N = 1$ chiral superfield, φ, known as the chiral compensator. The chiral compensator appears in the superdeterminant of the vielbein through the expansion

$$E = E(H_m) \, \varphi \bar{\varphi} \,, \tag{C.1}$$

where H_m is the real prepotential superfield which contains the physical polarizations of the graviton and gravitino.

---

[12]Gauge and gravity loops are important for subdominant diagrams and at higher-loop order; we would need to generalize this analysis to accommodate them.

In superspace, the lagrangian for the relevant part of the coupled supergravity-matter system is [12], [18]

$$L = -3M_P^2 \int d^4\theta \, \varphi\bar{\varphi} \, e^{-K/3M_P^2} + \left(\int d^2\theta \, \varphi^3 W + \text{h.c.} \right) , \qquad (C.2)$$

where K and W are the Kähler potential and superpotential of the hidden- and observable-sector fields. Equation (C.2) is valid in a flat gravitational background, where $E(H) = 1$, so we assume that the cosmological constant is canceled by the Kähler potential (as in LHC models) or by a constant in the superpotential (3.3).

In addition to local supersymmetry, this lagrangian possesses a local super-Weyl-Kähler symmetry, under which the superpotential, Kähler potential and chiral compensator transform as follows,

$$\begin{aligned}
W &\rightarrow e^{-F/M_P^2} \, W \\
K &\rightarrow K + F + \bar{F} \\
\varphi &\rightarrow e^{F/3M_P^2} \, \varphi \\
\bar{\varphi} &\rightarrow e^{\bar{F}/3M_P^2} \, \bar{\varphi} .
\end{aligned} \qquad (C.3)$$

This is a symmetry of the "kinematics" – the torsion constraints of $N = 1$ supergravity. It is not, however, a real symmetry of the lagrangian after the fields have been fixed at their vacuum values, which are determined by their equations of motion and by the requirement of canonical Einstein gravity. The symmetry is crucial for the superspace formulation of the theory, so we explicitly retain it for all superfield calculations. It is this symmetry which leads to the usual component Kähler invariance, after eliminating the component auxiliary fields and rescaling the component metric [8].

Supersymmetry is spontaneously broken by the vevs of the F-components of the nonpropagating compensator and the physical hidden-sector fields. These vevs break the super-Weyl-Kahler invariance. Nevertheless, the compensator formalism permits calculations to be done in a supersymmetric and super-Weyl-Kahler-invariant manner. At the end of the calculation the compensator and the F-components of all other fields are eliminated through their equations of motion. This simplifies the calculation because one can use the superspace Feynman rules. Furthermore, one can find the supersymmetry-breaking vertices from their supersymmetric counterparts using the procedure which we now describe.

We will first assume that the lagrangian (C.2) is the classical lagrangian of the theory. We can then determine the tree-level vevs of the chiral compensator and the hidden-sector fields. The

lowest component of the chiral compensator is fixed by requiring that (C.2) yields the correctly normalized Einstein gravitational action

$$- 3M_P^2 \int d^4x d^4\theta \ E(H_m) \ = \ -\frac{1}{2}M_P^2 \int d^4x \, e\mathcal{R} \ + \ ... \ , \tag{C.4}$$

where we have restored the part of the supervielbein that contains the physical graviton and gravitino. If (C.2) is to yield (C.4), we must fix super-Weyl-Kähler gauge by requiring

$$\varphi\bar{\varphi} \ e^{-K/3M_P^2}|_{\theta=\bar{\theta}=0} \ = \ 1 \ . \tag{C.5}$$

Fixing this gauge is equivalent to Weyl rescaling the metric in the component theory.

To find the F-components of the fields, we then compute the superfield equations of motion from (C.2),

$$-\frac{1}{4}\bar{D}^2\left(\bar{\varphi} \ e^{-K/3M_P^2}\right) \ = \ \varphi^2 W$$
$$-\frac{1}{4}\bar{D}^2\left(\varphi\bar{\varphi} \ e^{-K/3M_P^2}K_i\right) \ = \ -\varphi^3 W_i \ . \tag{C.6}$$

Using the transformation law (C.3) it is easy to show that these equations (C.6) transform covariantly under super-Weyl-Kähler transformations. Taking the lowest component of the first equation in (C.6), and fixing the lowest component of φ by requiring

$$\varphi|_{\theta=0} \ = \ \bar{\varphi}|_{\bar{\theta}=0} \ = \ e^{K/6M_P^2}|_{\theta=\bar{\theta}=0} \ , \tag{C.7}$$

we find the following result for the vevs of the chiral compensator superfields,

$$\varphi \ = \ e^{K/6M_P^2}\left[1 \ + \ \theta^2\left(e^{K/2M_P^2}\frac{\bar{W}}{M_P^2} \ + \ \frac{K_i F^i}{3M_P^2}\right)\right]$$
$$\bar{\varphi} \ = \ e^{K/6M_P^2}\left[1 \ + \ \bar{\theta}^2\left(e^{K/2M_P^2}\frac{W}{M_P^2} \ + \ \frac{K_{i*}F^{*i}}{3M_P^2}\right)\right] \ . \tag{C.8}$$

In a similar fashion, using eqs. (C.6), we can find the solution for the matter F-terms,

$$F^i \ = \ -e^{K/2M_P^2}K^{ij*}D_{j*}\bar{W} \ \equiv \ -e^{K/2M_P^2}\bar{W}^i \ ,$$
$$F^{*j} \ = \ -e^{K/2M_P^2}K^{ij*}D_iW \ \equiv \ -e^{K/2M_P^2}W^{j*} \ . \tag{C.9}$$

These tree-level vevs can be inserted into one-loop graphs to determine the quadratically divergent component operators. They must be corrected at higher-loop order.

The choice (C.7) explicitly breaks the component Kähler invariance, and indeed, the solutions (C.8) transform noncovariantly. However, the final component action is still Kähler-invariant because it is obtained from a super-Weyl-Kähler invariant expression. This can be verified explicitly by substituting (C.8), (C.9) into the lagrangian (C.2). We have checked that this procedure gives the standard Kähler-invariant formula for the supergravity scalar potential (3.2).

To compute the loop corrections to the lagrangian (C.2) in the classical background (C.8), we expand the matter superfields to second order in the fluctuations Φ around their vevs:

$$
\begin{aligned}
L \;=\; L_{class} \;&+\; \int d^4\theta \left(\varphi\bar\varphi\, e^{-K/3M_P}\, K_{ij*}\right)_{class} \Phi^i \Phi^{+\,j} \;+\; ... \\
&+\; \left(\int d^2\theta \left(\frac{1}{2}\, \varphi^3\, W_{ij}\right)_{class} \Phi^i \Phi^j \;+\; \text{h.c.}\right) .
\end{aligned} \tag{C.10}
$$

In this expression, the dots denote terms suppressed by additional powers of M_P. For notational simplicity, we assume that all vevs are smaller[13] than M_P.

The Feynman rules that follow from this expansion are easily obtained from the flat-space super-Feynman rules. The chiral vertices carry an additional factor of φ^3, and the chiral field propagators are of the form

$$
\langle \Phi^i(p,\theta)\Phi^{+\,j}(p,\theta')\rangle \;\simeq\; \left(\frac{e^{K/3M_P}}{\varphi\bar\varphi}\, K^{ij*}\right)_{class} \frac{\delta^4(\theta-\theta')}{p^2} . \tag{C.11}
$$

Note that the above expression is not the exact propagator in the superfield background [19]. We have omitted terms where the supercovariant derivatives act on the background superfields. These terms reduce the degree of divergence and do not contribute to the leading divergences [20].

After applying the above super-Feynman rules, the logarithmically divergent part of the graph from Fig. 1 is easily seen to be

$$
\frac{\log\Lambda^2}{32\pi^2}\, \int d^4\theta\, e^{2K/3M_P^2}\, \varphi\bar\varphi\, W_{ij} K^{il*} K^{jm*} \bar W_{l*m*} . \tag{C.12}
$$

This precisely equals (4.4). The same Feynman rules, when applied to the quadratically divergent two-loop supergraph of Fig. 2, yield the result (4.16) from Sect. 4.

---

[13]One might worry that the expansion (C.10) is not explicitly super-Weyl-Kähler invariant since the derivatives of W are not Kähler covariant. However, the expansion (C.10) is gauge equivalent to a reorganized expansion, obtained by redefining the chiral compensator: $\varphi = \varphi' \exp\left(-\log W/3\right)$. The whole expansion then only depends on the super-Weyl-Kähler invariant $G = K + \log W\bar W$ [18], and therefore manifestly preserves the super-Weyl-Kähler invariance. The Feynman rules, however, are more cumbersome and for simplicity we prefer to use the expansion (C.10). We have explicitly checked that the divergent contribution to the effective action obtained by using this expansion is equivalent to ours, after using the superfield equations of motion for the background superfields.

D   The Two-Loop Diagram with a Higher-Derivative Regulator

In this Appendix we describe the calculation of the quadratically divergent supergraph of Fig. 2, using a supersymmetry-preserving higher-derivative regulator. As explained in the previous Appendix, we can reduce the calculation to that of a rigid supersymmetric theory in the background of the hidden-sector and chiral-compensator fields. Hence it is sufficient to employ a rigid version of the higher-derivative regulator. (For a recent discussion, see Ref. [21]).

To regulate the ultraviolet behavior of the chiral-field propagators, we add to their lagrangian the following term [15]:

$$
L_{regul} = \int d^4\theta \left(\varphi\bar{\varphi}\, e^{-K/3M_P}\, K_{ij*} \right)_{class} \Phi^i f(\frac{\Box}{\Lambda^2})\Phi^{+\,j} \ . \tag{D.1}
$$

The chiral propagator then becomes:

$$
\langle \Phi^i(p,\theta)\Phi^{+\,j}(p,\theta')\rangle = \left(\frac{e^{K/3M_P}}{\varphi\bar{\varphi}}\, K^{ij*} \right)_{class} \frac{\delta^4(\theta-\theta')}{p^2(1+f(p^2/\Lambda^2))} \ . \tag{D.2}
$$

In a theory without gauge fields, this procedure is sufficient to regulate all divergences [15].

To regulate the two-loop graph from Fig. 2, it suffices to take $f(x) = x$, as we show below. With this choice the integral (4.14) becomes

$$
\begin{aligned}
I &= \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} \frac{1}{(k^2+k^4/\Lambda^2)\,(q^2+q^4/\Lambda^2)\,((k+q)^2+(k+q)^4/\Lambda^2)} \\
&= \frac{\Lambda^2}{512\pi^4} \int_0^\infty dx \int_0^\infty dy\, \frac{\sqrt{(1+x+y)^2-4xy}-|x-y|-1}{(x^2+x)(y^2+y)} \ .
\end{aligned} \tag{D.3}
$$

The infrared structure of this integral is clearly the same as that of the integral (4.14) with the hard cutoff, so (D.3) also does not contain logarithmic terms. To investigate the ultraviolet behavior, it is convenient to reduce I to a one-dimensional integral

$$
I = \frac{\Lambda^2}{512\pi^4} \int_0^\infty dx\, \frac{Q(x)}{x^2+x} \ , \tag{D.4}
$$

where

$$
\begin{aligned}
Q(x) =\ & -2\,x\,\log(x) + 4\,\log(1+x) + 4\,x\,\log(1+x) \\
& +\sqrt{4\,x+x^2}\,\log(\frac{-x+\sqrt{4\,x+x^2}}{3\,x+x^2+(1+x)\,\sqrt{4\,x+x^2}}) \ .
\end{aligned} \tag{D.5}
$$

At infinity, the integrand in (D.4) behaves as $x^{-3} \log x$, and at zero it is $\sim \log x$. Therefore the integral converges in the UV and the IR, so it can be taken numerically. The result is

$$I = 2.84 \frac{\Lambda^2}{512 \pi^4} \; . \tag{D.6}$$
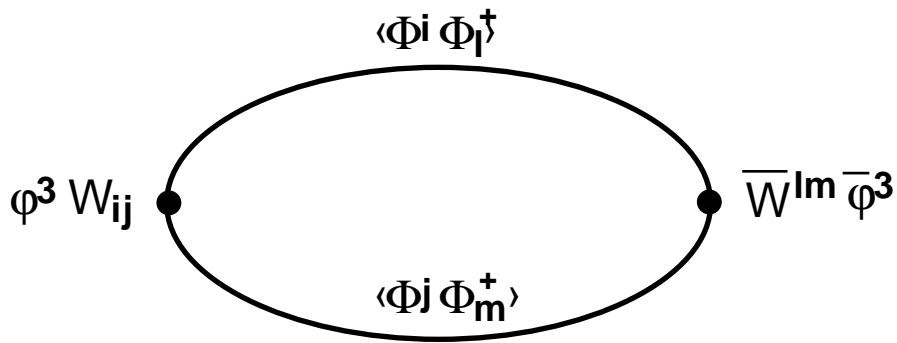
Fig. 1: One-loop logarithmically divergent
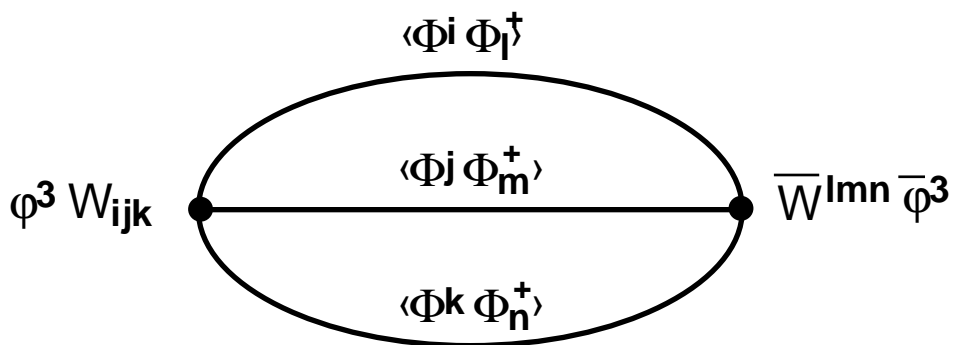contribution to the Kähler potential

Fig. 2: Two-loop quadratically divergent
contribution to the Kähler potential